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Nash balance and estimates of the chances of future events occurring

Equilibrio de Nash y estimaciones de las posibilidades de ocurrencia de eventos futuros

Equilíbrio de Nash e estimativas das chances de ocorrência de eventos futuros

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Abstract

Decisions are made in dynamic or stable environments, in situations involved in greater or lesser risk. The problems are then solved with the application of reason, subjectivity of the decision-maker, or the combination of both approaches, always depending on anticipations about the context of the decision in the face of time availability for such a decision. This article presents a proposal for the application of the Nash Equilibrium theory in situations of quick decisions. Based on two hypothetical cases, adaptations, adjustments and complements to the theory of balance are presented that elucidate the feasibility of applying this theory in situations of quick decision-making, in dynamic environments in the Defense sector. In conclusion, adaptations and proposals with potential for improvement of decisions in the Defense sector are made, in situations that require this type of deliberation.

Keywords: Quick decisions. Uncertainties. Defense Sector. Nash Equilibrium.

RESUMEN

Las decisiones se toman en entornos dinámicos o estables, en situaciones de mayor o menor riesgo. Los problemas se resuelven entonces aplicando la razón, la subjetividad de la persona que toma la decisión, o combinando ambos enfoques, siempre en función anticipada del contexto de la decisión contra la disponibilidad de tiempo para tal decisión. Este artículo presenta una propuesta para aplicar la teoría del Equilibrio de Nash en situaciones de decisión rápida. A partir de dos casos hipotéticos, se presentan adaptaciones, ajustes y complementos a la teoría del equilibrio que dilucidan la viabilidad de aplicar esta teoría en situaciones de toma de decisiones rápidas, en entornos dinámicos del sector de Defensa. En conclusión, las adaptaciones y propuestas con potencial para perfeccionar las decisiones en el sector de defensa se hacen en situaciones que requieren este tipo de deliberación.

Palabras clave: *Decisiones rápidas. Incertidumbres. Sector de Defensa. Equilibrio de Nash.*

RESUMO

As decisões são tomadas em ambientes dinâmicos ou estáveis, em situações envoltas em maior ou menor risco. Os problemas são, então, resolvidos com a aplicação da razão, da subjetividade do decisor, ou com a conjugação de ambas as abordagens, sempre em função de antecipações sobre o contexto da decisão defronte à disponibilidade de tempo para tal decisão. Este artigo apresenta uma proposta de aplicação da teoria do Equilíbrio de

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The acronyms and abbreviations contained in this article correspond to the ones used in the original article in Portuguese.

Nash em situações de decisões rápidas. A partir de dois casos hipotéticos, apresentam-se adaptações, ajustes e complementos à teoria do equilíbrio que elucidam a viabilidade da aplicação dessa teoria em situações de tomadas de decisões rápidas, em ambientes dinâmicos no setor de Defesa. Concluindo, são realizadas adaptações e propostas com potencial para aperfeiçoamento das decisões no setor de Defesa, em situações que exijam esse tipo de deliberação.

Palavras-chave: *Decisões rápidas. Incertezas. Setor de Defesa. Equilíbrio de Nash.*

1 INTRODUCTION

This paper aims to reinforce the arguments in favor of expanding the use of John F. Nash's equilibrium theory in estimates about the chances of occurrences of future events, especially in situations where there is scarcity of time for decision and where System 1 (intuitive thinking) (STANOVICH; WEST, 2000) can be incorporated into the employed decision-making method (KAHNEMAN, 2011).

Such reinforcement in favor of Nash Equilibrium theory will be initiated through two hypothetical experiments, namely: the SISFRON Case (MOURA; SILVA, 2006), this being an adaptation of the Boeing versus Airbus case (BESANKO; DRANOVE; SHANLEY, 2000), and the "Abbottabad Case" (FRIEDMAN; ZECKHAUSER, 2014).

It is important to note that the Boeing versus Airbus case presents solutions for choices in which profit is the beacon of the decision, while the cases used in this work as experiments of the methodology (Case SISFRON and Case Abbottabad) change the direction of the analyses, pointing more towards the national defense.

Once stated in the following section, the problem on screen, its objectives and the guiding hypothesis, the theoretical foundation concerning begins, that is: *(i)* the theory of decisions focusing on the immediate interest of the present work and *(ii)* Nash's Equilibrium theory. In the first item will be addressed ramifications of decision theories, such as quick decision trees. After reviewing the literature, the two hypothetical experiments, discussions and conclusions are presented.

2 PROBLEM, OBJECTIVES AND HYPOTHESIS

The problem addressed in this article is that of quick decisions. Quick Decisions (GIGERENZER; GOLDSTEIN, 1996) are part of the routine of one of the professions involved with the defense affairs, the military profession. One of the corollaries of quick decisions is the necessary combination, in series or in parallel, of the systems of intuitive (system 1) and structured (system 2) thought.

The main objective of this work is to highlight the plausible application of Nash Equilibrium theory in situations such as those that were enunciated in the previous paragraph. As secondary objectives, we seek an increase in the strategies for the improvement of decisions, *lato sensu*, by the calibration of decision methods classified as hybrids (VIANNA 1982, 1989; SILVA, 2000), in which estimates of subjective probabilities present weight equal to, or greater than, the distributions of objective probabilities (STEVENSON, 1986).

The assumption of this work is that the Nash Equilibrium theory, in addition to presenting itself as plausible of application in situations of quick decisions, should also be classified as a preferred instrument for the solution of trees of quick decisions, in which uncertainties need to be consistently quantified, although there is not sufficient decision time for the total structuring, that is, to estimate the chances of all variables involved in the decision and the appropriate probability distributions. So that, even before the discussion of the results of the work, and still distant from its conclusions, it is necessary to affirm that the coherent definition of the term applicability, previously used, refers only to the initial indications about the best course of actions to be followed in the quick decision to be taken.

3 LITERATURE REVIEW

3.1 Decision theory and its aspect of quick decisions

The theory of decisions can be classified into two major branches: the branch of decisions made in conditions of greater control and greater predictability, and the branch of decisions made in conditions of less control and greater uncertainty. Since uncertainty – or the degree of ignorance in its strongest sense – is one of the characteristics of the environment in which contemporary Defense operations develop, it is under the second branch that this work presents itself.

So that the decisions in conditions of uncertainty involve the study of probabilities to reduce the degree of ignorance about the origin of threats. Both objective and subjective probabilities.

With regard to objective probabilities, we will refer to the "frequencyist" tradition - future estimates based on past frequencies – specifically the Bayesian decision theory (STEVENSON, 1986; SILVA, 2004).

The Baysean's theory of decision- making, published after the death of its author Thomas Bayes in 1763, is a criterion for estimating probabilities in which estimates are renewed based on new evidence related to the estimated event. A doctor can review the probability estimates that a patient – even though he or she is in a specific risk group for a particular type of cancer, will really have cancer, after presenting negative results in imaging tests for the disease.

With regard to subjective probabilities, we will take as reference the quick decisions, in which the weight of subjective opinions is relevant and must be coherently calibrated (VIANNA, 1982, 1989; SILVA, 2000). All under the wrapper of the scarcity of decision time and limitation of the information available for decision making.

Quick decisions are defined as those that are made in situations of time scarcity, uncertainties regarding the best choices and consequences resulting from choices presenting potential for positive/negative impact of significant degree (KLEIN, 2000; GIGERENZER, 2002; GAWANDE, 2010; KAHNEMAN, 2011).

We present as beacons of our choices two systems of thought (STANOVICH; WEST, 2000). The system 1, intuitive, which is faster and more affluent for all of us, but is based on our repertoire of previous experiences. The more previous experience, the greater to chance of successfully applying system 1. And system 2, structured and based on the postulates of instrumental logic, which depends on greater time for a successful execution. It is important to reinforce that for the most coherent use of system 1, the decision agent needs to have a lot of experience in the subject, so that he can decide based on his intuition (GIGERENZER, 2009).

Stable environments are not those devoid of uncertainties and risks (threats), but those in which such threats are well known. We can exemplify these environments as those that are part of the activities of doctors and pilots of airplanes, because, although they initially convey the idea of instability, one has the knowledge of the numerous threats and risks routine. And in these environments, where decisions must be made in the short term, quick decision trees[1] are the most promising tool.

One of the most accepted branches in decision theory, for the design of trees of quick decisions, in which new information can change the "design" of future estimates is the Bayesian theory of decision (GIGERENZER, 2002).

3.2 Nash Balance Theory

The Nash Equilibrium theory presents as the main axis of argumentation the idea that two agents will not reach the ideal level in their objective variables if each maintains the strategy of maximizing their preferences.

One of the most recurrent experiments used to illustrate this situation is the so-called "prisoner's dilemma"[2].

Nash's equilibrium theory presents the following mathematical notation: consider a game *(S, f)* with *n* participants where *Si* is the set of possible alternatives for each participant *i, S=S1X S2...X Sn e f=[f1(x), ..., fn(x)]* is the *payoff* function, or the function of possible consequences. So, the balance predicted by Nash would be in the $x^* \in S$ set, that is, $\forall i, xi, \in Si, xi \neq x^*i : fi(x^*i, x^*\text{-}i) \geq fi(xi, x^*\text{-}i)$.

Besanko et al. (2000) presented a rather interesting hypothetical example of the application of the Nash Equilibrium theory involving a dispute between the two largest passenger aircraft manufacturers, Boeing and Airbus. The basic decision to be made in this case boils down to two inputs[3], that is, launch or not launch the large capacity aircraft, thus resulting in matrix of four inputs. Example in which it was demonstrated that the balance would occur in the situation where the two manufacturers would launch their *wide-body* aircraft, which in fact occurred of a few years after the hypothetical example was presented. Table 1 shows the *payoffs* of the hypothetical situation suggested by Besanko et al. (2000), in which in the hachured quadrant appears the Nash equilibrium.

Chart 1: Case Boeing versus Airbus.

	AIRBUS DOES NOT LAUNCH	AIRBUS LAUNCHES
BOEING DOES NOT LAUNCH	$18, $18	$15, $20
BOEING LAUNCHES	$20, $15	$16, $16

Source: Adapted from Besanko et al. (2000). Amounts in millions of dollars annually, and the first values of the line correspond to Boeing's net profits, the second amounts correspond to Airbus's net profits.

[1] For more details on the theoretical rationale and application of quick decision trees, check: SILVA, 2013.

[2] The author was John Nash's teacher, Albert William Tucker (1905-1995).

[3] The matrix logic respects axiom n^2, where n is the number of options existing for the basic decision.

4 HYPOTHETICAL EXPERIMENTS

As we said earlier, the methodology of this work will be developed through two hypothetical experiments, the SISFRON Case (MOURA; SILVA, 2006) and the Abbottabad Case (FRIEDMAN; ZECKHAUSER, 2014).

4.1 SISFRON Case

The SISFRON Case (MOURA; SILVA, 2006) is an adaptation of the Boeing *versus* Airbus case (cited above) in which the Nash Equilibrium theory is used, plus the rollback method of decision[4], whose main strategy is the optimization by preview of the possible choices of the opponent before each party makes its choices. This is done by solving a quick tree of decision backwards, in order to anticipate the decisions of the opponent, and it is unnecessary to argue about its practical applicability in the defense sector. It is also important to observe the reducing character of decision options, without having to resort to statistical resources, such as factor analysis (HAIR et al., 1987). The method also allows the identification of the dominance[5] existing among the alternatives, without it being necessary to resort to any mathematical or preference notation (GOODWIN; WRIGHT, 2009).

Another important aspect of the adaptation performed by the authors is the fact that it was elaborated to illustrate a hypothetical situation of conflict between two countries, the "Alpha Country" and the "Bravo Country", which would have as alternatives a matrix of three entries, not two anymore, which leads to a picture with a range of nine alternatives. The basic decision involves the alternatives "not investing in the armed forces"; "investing in border protection"; and "invest in the armed forces".

The Alfa Country borders the sea, thus having great maritime potential, such as deep-water oil exploration and access to important export markets. Bravo Country borders the Alpha Country, but does not have access to the sea. The Bravo Country has in oil exploration its main source of wealth and could substantially reduce its freight costs, if the flow of its production occurred by sea. Both are developing countries and have a history of being peaceful nations. The two don't have the military forces, only a central force with a police role and responsible for maintaining the sovereignty.

Although there is no history of armed conflicts between the Alpha and Bravo Countries, these nations began to present a fiercer posture of relationship, due to strategic interests that each has in the natural resources of the other.

From this context, the Alpha Country begins to analyze the need to make investments in the creation of its armed forces, foreseeing possible intensification of relations with its neighbor or even possible war conflicts. The Bravo Country, through its intelligence services, has discovered there are indications of possible decisions of investments in armaments of the Alpha Country.

Thus, once all the possibilities of direct negotiation between the countries have been exhausted, even through multilateral bodies, the governments of both countries are in the midst of a complex decision, which presents – at least – the following alternatives for both sides: (i) not invest in armed forces, yet seeking to maintain good diplomatic relations with the neighbor; (ii) invest only in greater protection of its borders, demonstrating precautions and strong willingness to defend its sovereignty; (iii) massively invest in the development of the armed forces, preparing for open conflicts.

Table 2 presents an estimate – carried out by the Alpha Country – of the percentages of Gross Domestic Product (GDP) growth of each country, in each of the alternatives discussed above.

It is important to highlight two key aspects involved in the solution of this quick decision tree. First of all, in the central quadrant, hachured in gray, the point at which Nash's balance would be reached. Secondly, it is worth clarifying some questions related to the optimal decision for the "Alpha Country", by the *rollback method*. For this, consider the decision tree (Figure 1).

It should be noted that the best strategic option for the Alpha Country would be to invest in armed forces, since this alternative has the highest GDP value (4.8%) for the country, once it is made the disposal of the weakest alternatives for the Bravo Country.

[4] Although the *rollback method* is elucidated in this section, for further investigation we suggest the work of Goodwin and Wright (2009).

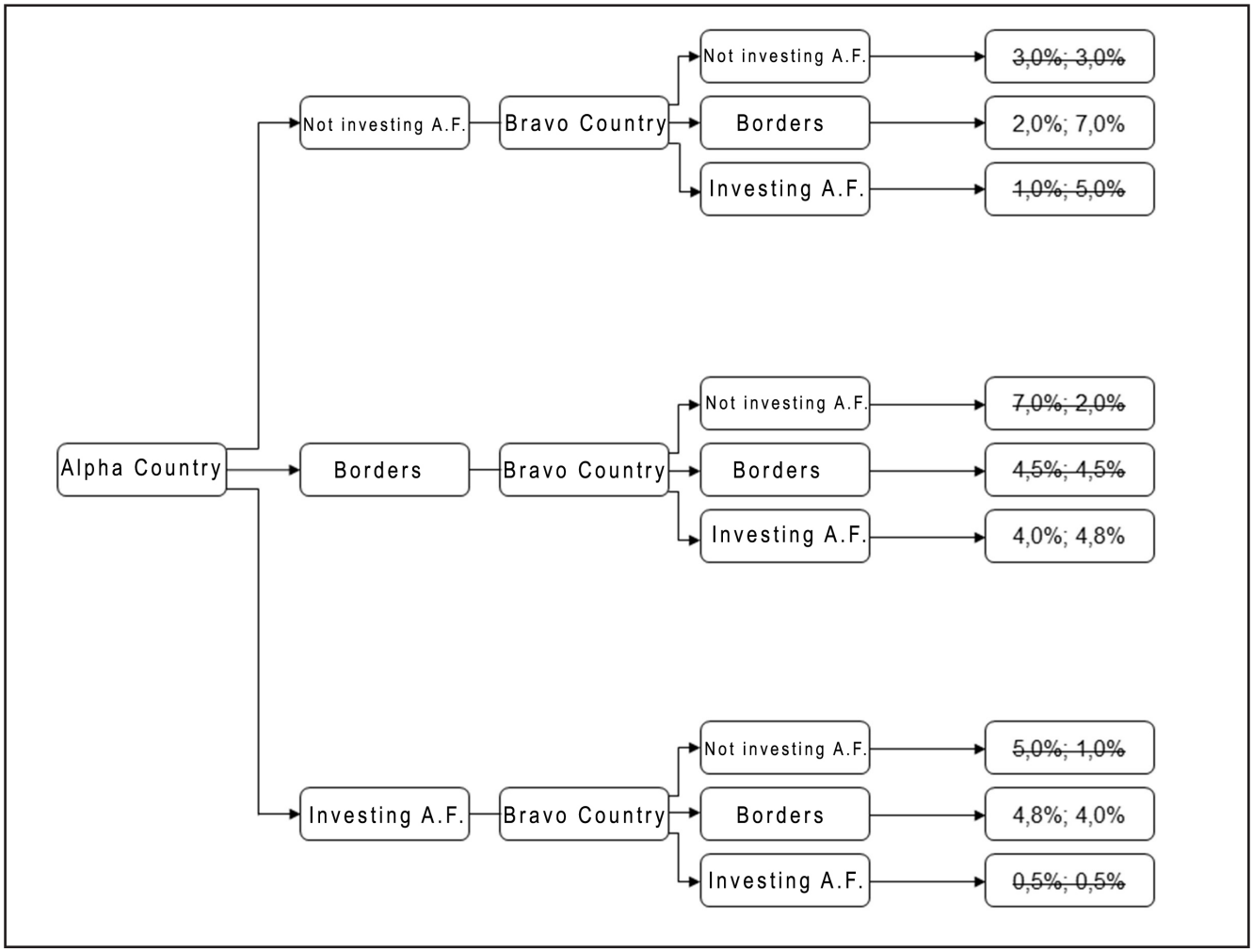
[5] Figure 1 shows the scratched values, representing their submission to the dominant values. In this case, the dominant values for the Brave Country, once the *rollback method* has been applied.

Chart 2 – Estimates performed by Alpha Country.

	Bravo Country: Not investing in Armed Forces	**Bravo Country:** Investing in border protection	**Bravo Country:** Investing in Armed Forces
Alpha Country: Not investing in Armed Forces	3,0%; 3,0%	2,0%; 7,0%	1,0%; 5,0%
Alpha Country: Investing in border protection	7,0%; 2,0%	4,5%; 4,5%	4,0%; 4,8%
Alpha Country: Investing in Armed Forces	5,0%; 1,0%	4,8%; 4,0%	0,5%; 0,5%

Source: The authors (2017). GDP growth percentage values, the first values of each row referring to the Alpha country and the second values referring to the country Bravo.

Figure 1 – Quick decision tree by *rollback method.*



Source: The Authors (2017).

In our understanding, both the Nash Equilibrium theory, developed from quick decision trees, and the *rollback optimization method* should, for their simplicity and empiricism, be included in the list of probability estimation techniques used in military operations in Brazil. Or, as is agreed in the title of this work, the techniques of estimating the chances of occurrence of future events in the scenarios elaborated by our armed forces.

So, let's move on to the second of the hypothetical experiments proposed for this work.

4.2 Abbottabad Case

Friedman and Zeckhauser (2014), referring to the operation to capture and kill Osama Bin Laden, which took place in Abbottabad in April 2011, claim that then-U.S. President Barack Obama expressed his discomfort with the situation that had taken place in the crisis office, moments before the military mission was carried out.

This discomfort would result from different estimates of probabilities that were being elaborated by the different members of the crisis cabinet, regarding the chances that the terrorist was even inhabiting the Pakistani residence that should be invaded. And whose final decision would be up to the president.

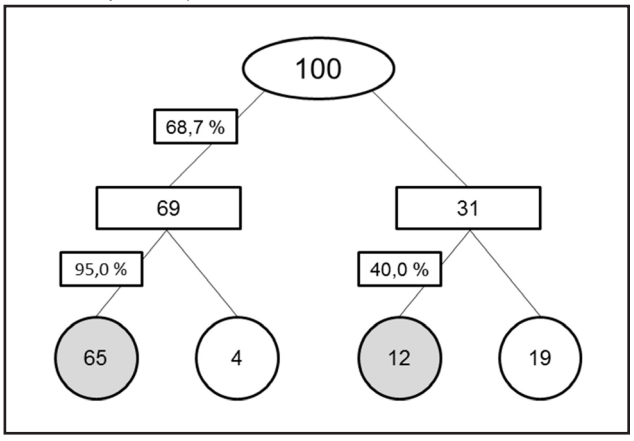
According to Friedman and Zeckhauser (2014), the crisis cabinet was then composed *(i) of the White House Red Team*, a team that plays the role of "devil's advocate"; *(ii)* by the Director of Intelligence of the U.S. Congress, Congressman Michael Morell; *(iii)* by CIA leader Mark Bowen; and *(iv)* by the other members of the crisis cabinet, such as the Secretary of State and other national defense agents.

The estimates of probabilities drawn up regarding the chances that Bin Laden was actually inhabiting that residence would have been: 40% in the opinion of the *Red Team*; 60% in the opinion of the Deputy Director of Intelligence; 95% in the CIA leader; and 80% in the other members of the crisis cabinet. Then the president would have declined to be confused not knowing what to do with all these estimates. Take a simple average or a weighted average? And what about the nuances behind each of the considerations performed, given the different professional profiles existing in that office? How to compare the most reliable estimate with the new evidences?

Exactly in the sense of contributing to these questions, but assisting in the search for answers, at least by evidencing the practical applicability of some tools, such as Nash Equilibrium and quick decision trees, is that we understand the justification of these hypothetical experiments. This time, in this section, we propose a variant of the Equilibrium theory applied to fast decision trees, such as the natural frequencies of Gigerenzer (2002). Used for the renewal of the odds based on the evidence, within the theoretical framework of the Bayesian tradition.

For this we suggest Figure 2, which should be interpreted according to the postulates of Gigerenzer's natural frequencies (2002 apud SILVA, 2004). According to the theory of natural frequencies, we would have the average of the opinions of the members of the crisis cabinet as a basic reference rate, 68.7%. The most favorable estimate, that Bin Laden was in the house – and more grounded in terms of subjective probability (VIANNA, 1982, 1989) – is taken as a positive support for the decision to be made (95%). And the most pessimistic estimate (40%) is taken as false positive (Bin Laden would not be in the house).

Figure 2 - Quick decision tree by the Gigerenzer method(2002) (of natural frequencies).



Source: The authors (2017).

Explaining Figure 2, there is 68.7% of the average of opinions that, for calculation adjustments, will be considered 69%. That way, we'd have a 69% chance that Bin Laden was in place and a 31% chance that he wasn't. Thus, according to the postulate, we will use 95% (most favorable estimate) for the average of opinions and 40% (more pessimistic estimate) for the probability of Osama not being in the house. The result found would be a=65 (95% of 69), and b=12 (40% of 31).

Thus, in order to achieve the estimate that would be presented to the U.S. president regarding the chances that Bin Laden was in the residence at the time of the invasion, without disregarding the opinions of any of the members of the crisis cabinet, according to Gingerenzer's (2002) postulates, would be given by the formula (a/a+b). It is considered a=65 and b=12

(the highest probabilities found in each bias, positive and pessimistic), resulting in a single one-off estimate that would be presented to the president, of 84.4% (a/a+b) probability that Osama bin Laden was in the target residence. That is, the probability to be transmitted to the decision-maker, in this case the President of the United States, would be 84.4% of Osama Bin Laden being in the house to be occupied. Friedman and Zeckhauser (2014) refer to the importance of indicating specific estimates to decision-makers.

5 DISCUSSION

The central issue, on which we seek to reflect, is about rapid decision-making, with the adjacent finality of introducing possible approaches to the decision-making process associated with Defense issues and military operations. It started from the premise that military decisions are routinely taken under both environmental restrictions (environment of uncertainty) and time (scarcity of this). This is because, considering a situation of need for the use of military forces, decision-makers may not know in detail the scale of the threats in parallel with the need for response in a short period of time. It was estimated that the adoption of specific techniques for modeling the decision-making process may favor decision-making itself, as they cooperate with a better integration of the cognitive processes involved. In other words, it was estimated that certain techniques favor the integration of the decision-making process based on personal and professional experience with the process based on logic and rational calculation.

It was also succinctly verified that the techniques are viable, that is, they are possible to be used in decision contexts typical of the Defense area, as well as with the support of these, it is possible to reach structured solutions. This statement is ratified by observing the cases studied, in which they proved possible to be solved efficiently with the use of the techniques proposed in this work.

It was also found that the techniques briefly described are of potential interest for planning work, especially at the tactical level, simultaneously less complex and demanding of faster decisions, or, at any level, when the central issue is the decision-making with insufficient cognition and quickly.

Reflecting based on the brief case studies, it is possible to realize that analytical complexity grows exponentially. Each new intervening variable added to the process significantly increases the estimation process and, consequently, the decision-making. It means that there is a need to keep a reduced number of variables under examination in order to better take advantage of the potential of the techniques without losing sight of the central element that has been studied: the quick decisions.

6 CONCLUSIONS

This work presented some hypothetical applications of Nash's equilibrium theory, with adaptations that demonstrated it is possible to confer practical applicability to operations and tactical decisions in the Defense sector.

Initially, as problematic and in order to make initial clarification and understanding, some questions with regards to quick decisions, in which the differentiation between decision systems 1 and 2, stands out, respectively, as well as the applications of these systems in the decisions of military issues.

As the main objective of evidencing practical application of Nash Equilibrium theory in quick decisions in Defense issues, it is observed that, due to the simulated situations, this theory proved to be effective, contributing to the continuous study and possible real applications by the Armed Forces and decision-makers in the area of Defense.

It is observed the confirmation of the assumptions that the Nash Equilibrium theory, besides presenting itself as plausible of application in situations of quick decisions, should also be classified as a preferred instrument for the solution of trees of quick decisions. In the case of SISFRON, applying the Nash Equilibrium theory in a hypothetical dispute between two countries, the correctness of the application of the theory and its simulation is possible, since the Brazilian Armed Forces began to invest in the protection of their borders, which is confirmed with investments in the SISFRON program and the growing military operations in border areas. Such decision to invest in the protection of its borders was also the most likely indication pointed out by Nash Equilibrium theory.

The Abbottabad Case, on the other hand, contributed to evidence the practical applicability of tools in decision-making assistance, presenting the natural frequencies of Gingerenzer (2002) as a variant of the equilibrium theory applied to rapid decision trees. Thus, analyzing the proposed results and knowing the real story related to the fact narrated, it was verified that this could have been another effective tool to assist the decision-maker.

Thus, we can consider that the analysis of the estimates of the chances of occurrences of future events, based on the theories and assumptions presented in this work, can be tools of great value for decision-makers in the area of Defense, due to the characteristics and significance of the issues involved.

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